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Model Question Paper of
M.Sc Sem II
Paper Code - CC Math 203.
(Differential Geometry and Tensor Analysis)

Short Answer type Questions.

1) Establish the Serret-Frenet Formulae at a point of a space curve.

2) For the curve $x = a(3u - u^3)$, $y = 3au^2$, $z = a(3u + u^3)$ show that the curvature and torsion are equal.

3) Find the osculating plane, curvature and torsion at any point of the curve $x = a\cos 2u$, $y = a\sin 2u$, $z = 2a\sin u$.

4) Prove that the curve
$x = au$, $y = bu^2$, $z = cu^3$ is a helix iff $3ac = \pm 2b^2$.

5) If R is the radius of the spherical curvature, show that
$$R^2 = \left|\frac{t \times t''}{K^2 T}\right|^2$$

(6) ~~For the~~ Calculate the fundamental magnitudes for the right helicoid given by $x = u\cos v$, $y = u\sin v$ and $z = cv$.

(7) Calculate the fundamental magnitudes for the Monge's form of the surface $z = f(x, y)$.

(8) If A^{pq}_{r} and B^{pq}_{r} are tensors, then their sum and difference are tensors.

(9) Define Inner product of two tensors. Show that the inner product of the tensor A^{p}_{r} and B^{qs}_{t} is a tensor of rank three.

(10) Prove that the necessary and sufficient condition that the two vectors $\vec{\lambda}$, $\vec{\mu}$ at O be orthogonal is

$$g(\vec{\lambda}, \vec{\mu}) = 0.$$

Long answer type Questions

1) Obtain expressions for radii for Curvature and torsion at a point of a given curve $\gamma = f(s)$, where s being the arc length.

2) Find the radii of curvature and torsion of the helix
$$x = a\cos u, \quad y = a\sin u, \quad z = au\tan\alpha.$$

3) Find the equations of the Principal normal and of the osculating plane at any point of the curve given by equations
$$x = 4a\cos^3 u, \quad y = 4a\sin^3 u, \quad z = 3c\cos 2u.$$

4) Find the equation of the osculating sphere and osculating circle at (1, 2, 3) on the curve
$$x = 2t+1, \quad y = 3t^2+2, \quad z = 4t^3+3.$$

5) Define helix. Show that a necessary and sufficient Condition for a curve to be helix is that the ratio of the Curvature and the torsion is Constant.

(6) A Curve is drawn on a parabolic cylinder so as to cut all the generators at the same angle. Find expressions for the curvature and torsion.

(7) Define Bertnard Curves. Show that the torsion at corresponding points of two Bertnard curves have the same sign, and that their product is const.

(8) State and prove Quotient Law of tensor.